MAPPING CHAOTIC DYNAMICAL SYSTEMS INTO TIMBRE EVOLUTION

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ABSTRACT

Until now many approaches have been used to transform in music and sounds time series produced by dynamical systems. Mainly these approaches can be divided in two categories: high level, finalized to melodic pattern generation, low level, in which dynamical systems are used to generate sound samples. In the present work we are going to present a new approach realizing a mapping at an intermediate level between the two previous mentioned in which we use chaotic systems to control the parameters of a sound synthesis process.

1. INTRODUCTION

Since 80s, chaos and fractal geometry have strongly affected the development of new fields of musical research, with the use of non-linear dynamic systems for musical purpose. By a different point of view, many researches are focus on the examination of scientific dataset by using musical tools and techniques in order to enhance the understanding of complexities of the data. According with Witten [17], "scientific datasets are often large, complex, and beyond the scope of simple visual understanding"; moreover as Pressing [14] points out "probably the ear can follow four dimensions more readily" than two or three and then through an auditory rendering of a dataset it is possible to find more information about a complex time series than what is possible to do with a simple visualization.

Generally a system generating sounds or music by using complex dynamical systems can be divided in three main parts (see Table 1): a dynamical system generating a time series, a codification system allowing to translate the time series into musical and/or sound synthesis parameters and a synthesis engine transforming these parameters into music and/or sounds. In our opinion the most important aspect in defining such systems consists in the careful choice of the codification scheme that must be able enough to faithfully convey into auditory event the evolution of the time series generated by the dynamical system,

achieving, therefore, an auditory "visualization" of the

	In	Out
Generator	system's parameters	n-dimensional time
		series
Codification	n-dimensional time	musical and/or sound
	series	synthesis parameters
Synthesis	musical and/or sound	sounds and/or music
	synthesis parameters	

Table 1. A schematic representation of a system to generate sounds and/or music by using a dynamical system.

The codification scheme is often also called *mapping* highlighting that in most cases it consists in a function that maps every element in the dynamical system's space of phases to a unique element of the space of musical and/or sound synthesis parameters.

Two different approaches have been used for music generation through non-linear dynamic systems: translating a time series into melodic patterns [2, 3, 14, 17], or using the time series for generating audio samples [6, 7, 8]. In more recent years both Cellular Automata (CA) [4, 5, 12] and evolutionary method by means of Genetic Algorithm (GA) [11] have been also used to generate melodic patterns. The former approach, based on melodic pattern generation, is the most used one, from a musical point of view, it can be thought as a high level mapping because it is finalized to translate the time series into high level musical events like notes with a specific pitch, amplitude and duration. In this kind of scheme sound's characteristics are not take into account, in fact generally this codification systems produce MIDI files to be rendered into sounds by using any General MIDI device [3]. The latter approach instead can be thought as a low level mapping in which dynamical systems are used to generate directly the fundamental element of the sound: sound samples.

In the present work we are going to present a new approach realizing a mapping at an intermediate level between the two previous mentioned. We use chaotic dynamical systems to control the parameters of a sound synthesis process mapping the evolution of the system into timbre modification of the sound. This intermediate

level mapping allows the user, by means of a MIDI keyboard, to control only the high level musical parameters, like pitch and amplitude, while the sound generated by the synthesizer evolves according to the chaotic dynamical system.

The paper is organized as follows: Section 2 presents an introduction to use of *mapping* in musical applications. Section 3 explains the approach we have adopted to map the evolution of dynamical systems, characterized by one, two, and three dimensions, into timbre modifications. Section 4 concludes the paper illustrating new directions of this work.

2. MAPPING

The codification system in a simple configuration allowing melodic pattern generation can be realized through a transformation of the range of the time series generated by the dynamical system into a fixed range of MIDI notes. This can be easily achieved through a linear mapping [9]:

$$y = \frac{(d-c)}{(b-a)}(x-a) + c$$
 (1)

transforming the variable x, lying between a and b in the variable y falling between c and d. If x is continuous, then y will be continuous. Likewise, a discrete x yields a discrete y. In this scheme the choice of a pitch vocabulary is crucial for determining the kind of music that results from the generative process. Very different results would be obtained by using a completely diatonic collection, a whole-tone collection, a pentatonic collection, or an octatonic collection.

Mapping is also widely used in a completely different context in the design of digital musical instruments as an operator that expresses each point in sound parameters space as a function of the input control parameters. In the design of a control system for a sound synthesis device it is important to define a strategies to effectively map controller variables to sound synthesis parameters that associates data from the input device, a collection of discrete and continuous control variables, to a possibly different number of inputs accepted by the synthesis engine [1, 10, 13]. In this context mapping can be built up from a pointwise map associating particular input values with output values: when the performer does this, the instrument should sound like this. An interpolator then produces reasonable intermediate outputs for intermediate inputs, by this point of view mapping then plays a large role in defining the *feel* of the instrument.

Generally, mapping strategies are separated in three different classes:

- one-to-one: one input parameter is mapped to one synthesis parameter
- one-to-many: one input parameter is mapped to several synthesis parameters

- many-to-one: many input parameters are mapped to a single synthesis parameter.

By combining these classes, *many-to-many* mappings can be built.

Since it is often the case that a sound synthesizer has many more real-time inputs than a human can attend to simultaneously, it can be useful reduce the complexity of a large set of parameters. A High-Dimensional Interpolator (HDI) lets the performer control a large number of parameters with a much smaller number of control inputs.

The term *mapping* encompasses both the choice of "what to map to where": the association of control and sound synthesis points themselves and the choice of the entire regions of control and sound synthesis space. This *how* component of mapping determines the trajectories and thus influences the musical gestures that are possible with an instrument. In the case of a discrete control mapping is essentially an assignment of a specific function, while in the case of a continuous control the role of mapping is more involved [13]. A collection of N continuous control variables that are simultaneously accessible can be assumed as a continuous N-dimensional Euclidean space, in which case mapping refers to both:

- the pointwise association between points in an N-dimensional controller space and an M-dimensional space of sound synthesis parameters. This can be seen as the what of the mapping.
- the rules governing the association of control/sound points not explicitly mapped in a pointwise fashion, but rather the association of entire sub regions of the respective parameter spaces. That is, the how aspect of mapping strategies."

From a mathematical point of view mapping is essentially a continuous function from \Re^n to \Re^m for arbitrary integers n and m, where $1 \le n < m$.

A first approach consists in building mapping procedures as a combination of relatively simple matrix operations. Consider $X = \{x_1, x_2, ..., x_n\}$ a vector of size \mathbf{n} from the controller parameters space and $Y = \{y_1, y_2, ..., y_m\}$ a vector of size \mathbf{n} from the sound parameters space. A simple mapping operation can be defined through the relation:

$$Y = A * X + B \tag{2}$$

where A is a $m \times n$ matrix and B is a vector of size m. The $m \times n$ coefficients of the A matrix and the m elements of the vector B can be exactly determined by defining a series of n+1 examples

 $\{X_i, Y_i\}$, where $1 \le i \le n+1$, associating points in the control parameters space with points in the sound parameters space [1, 10].

We think that the methods mentioned above about mapping between control parameters and sound synthesis parameters can be fruitfully used also in controlling sound synthesis process by using chaotic dynamical systems.

3. TIMBRE MODIFICATION

Timbre can be defined as the quality of sound allowing distinguishing between two sounds of the same pitch and loudness. Timbre refers to the "color" or quality of sounds, it is therefore a complex phenomenon representing that attribute of the auditory sensation allowing to the listener to identify the sound source distinguishing it from many others. Perceptual research on timbre has demonstrated that the spectral energy distribution and the temporal variation in this distribution provide the acoustical determinants of our perception of sound quality.

While pitch, loudness, and duration can be "discretized" and ordered a long a one-dimensional scale, the same scale can not create for the timbre that is something having more than one dimension. In fact timbres of different sounds can differ simultaneously by different aspects, so it is apparent that timbre is multidimensional; the perception of timbre involves correlating a number of factors of the tone, including the nature of the attack, the harmonic content, and the tuning of the partials.

Many researchers have proposed different kind of timbre space representations both for timbre analysis and classification and for new timbre generation by means of sound synthesis. The basic idea is that by specifying coordinates in a particular timbre space, one could hear the timbre represented by those coordinates. If these coordinates should fall between existing tones in the space, we would want this interpolated timbre to relate to the other sounds in a manner consistent with the structure of the space. Wessel [16] created a twodimensional timbre space controlling an additive synthesis process. One dimension of this space was used to manipulate the shape of the spectral energy distribution, while the other dimension was used to control either the attack rate or the extent of synchronicity among the various components.

Of considerable concern to composers of computer music it to find the means for creating sounds that represent systematic transformations from one recognizable timbre to another. A number of analytic tools provide facilities for creating this kind of "sound morph" [9]. An interesting sort of transformation is morphing two sounds to produce a new sound having characteristics of the two originals.

In sound morphing, the original source timbres are obtained at the extrema of the morphing function and

hybrid sounds are obtained from intermediate values of the morphing function. In the next paragraphs we are going to show the timbre modification approach we have defined based on sound morphing between different kind of waveforms.

3.1. 1-D Application

The first application we present is based on a *one-to-many* mapping allowing controlling the spectral content of the generated sound by using only one control parameter. This kind of application can be used to translate in the auditory domain the evolution of one-dimensional dynamical systems like the well known logistic map.

A simple instrument has been created through an additive synthesis process by adding ten sinusoidal partials. The mapping operation allows controlling with only one parameter the amplitude of all partials in the generated sound creating a sound morphing between two well defined waveform. The mapping has been realized by using the approach mentioned in the previous paragraph imposing these two conditions:

- when the parameter is equal to 1 we have a configuration of the partials' amplitude producing a sawtooth waveform
- when the parameter is equal to 100 we have a configuration of the partials' amplitude producing a square waveform.

The saw tooth waveform can be generated using sinusoidal harmonic components with this amplitude distribution (Figure 1):

$$a_{k} = \begin{cases} -\frac{1}{k} & \text{if } k \text{ even} \\ \frac{1}{k} & \text{if } k \text{ odd} \end{cases}$$
 (3)

where k = [1,...,n] is the number of the partial, while the square waveform can be generated with this amplitude distribution (Figure 2):

$$a_k = \begin{cases} 0 & \text{if } k \text{ even} \\ \frac{1}{k} & \text{if } k \text{ odd} \end{cases}$$
 (4)

In this case the mapping equation (2) can be written as:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ y_{10} \end{bmatrix} = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ a_{10} \end{bmatrix} * x + \begin{bmatrix} b_1 \\ \cdot \\ \cdot \\ b_{10} \end{bmatrix}$$
 (5)

where \mathcal{X} is the control parameter defined in the range (1, 100) and the Y vector contain the variables associated to the partials' amplitude. Imposing the two conditions mentioned above is possible to find the values of the elements of the vectors A and B by resolving the system of 20 equations in the 20 unknown variables. By this way it's possible to find ten equations relating the control parameter with the synthesis parameters, in this application corresponding with the amplitude of each partial.

Using this mapping a polyphonic synthesizer, called TIMBRALIZER-1D¹, has been created. The control parameter of the synthesizer has been controlled by using the iteration of a logistic map [1, 2]:

$$x_{n+1} = \alpha x_n (1 - x_n) \tag{6}$$

where \mathcal{X} variable is defined in the (0, 1) interval. In order to avoid abrupt variations in the spectral content of the waveform, that can generate unwanted click and noises, we have used a linear interpolator in between the data generated by the logistic map and the timbre's control parameter allowing to obtain a smooth variation of sound's spectral content over the time.

The synthesizer has been provided of a MIDI interface in order to respond to a Note control by using a external keyboard. By this way the user can control high level musical parameters, like pitch and amplitude, playing notes on the keyboard, while the spectral content of the sound evolves according to the logistic map.

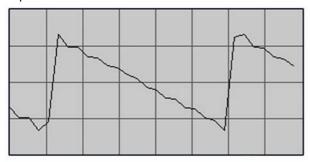


Figure 1. Sawtooth waveform generated when the control parameter is equal to 1.

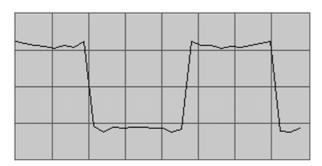


Figure 2. Square waveform generated when the control parameter is equal to 100.

Figure 1, 2, and 3 show the waveforms obtained for three different values of the control parameter. The tests we have performed show that this instrument allows to follow by an auditory point of view the evolution of the dynamical system. In fact it is possible to easily recognize, in a qualitative manner, the topologically different behaviors provided by the logistic map.

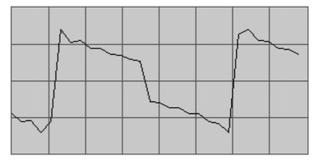


Figure 3. Intermediate waveform generated when the control parameter is equal to 50.

Moreover this instrument can be used to generate sounds with a very interesting dynamical evolution of the timbre, resembling the complexity of sounds generated by natural instruments. This interesting result of course depends on the behavior of the logistic map, which is characterized by three main topologically different evolutions:

- convergence to a fix point (α <2)
- periodic ($2 \le \alpha \le 3.5$)
- chaotic (3.5 \leq = α \leq 4).

The most interesting results can be obtained by choosing α parameter's values near 3.6 that correspond to the transition between periodic and chaotic behavior.

Moreover the timbric evolution of the sound can be modified also changing the time step used in the iteration of the logistic map. Changing this parameter it is possible to obtain both modulation like effect (like phaser or flanger) by lowering the time step under about 100 ms and complex timbre modification, evolving in different time scale, increasing the time step above this value.

3.2. 2-D Application

In the next paragraphs an application based on a *many-to-many* mapping allowing controlling the spectral content of the generated sound by using two control parameters is presented. We have tried to extend the previous one-dimensional application by using two different approaches in order to translate in the auditory domain the evolution of two-dimensional dynamical systems.

3.2.1. Tristimulus approach

In order to create a two-dimensional system allowing controlling the spectral content of the generated sound,

¹The instrument with sound examples can be downloaded from: http://www.musichenza.it/timbralizer/

an additive synthesis device in which the timbre is controlled by using a tristimulus scheme [15] has been implemented. The concept of tristimulus originates in the world of color, describing the way three primary colors can be mixed together to create a given color. By analogy, the musical tristimulus measures the mixture of harmonics in a given sound, grouped into three sections. The first tristimulus (T_1) measures the relative weight of the first harmonic; the second tristimulus (T_2) measures the relative weight of the 2nd, 3rd, and 4th harmonics taken together; and the third tristimulus (T_3) measures the relative weight of all the remaining harmonics, the follow equations can be used to evaluate the tristimulus indexes where H is the number of harmonics in the sound:

$$T_1 = \frac{a_1}{\sum_{h=1}^{H} a_h}$$
 (fundamental frequency) (7)

$$T_2 = \frac{a_2 + a_3 + a_4}{\sum_{h=1}^{H} a_h}$$
 (2nd 3rd 4th partials) (8)

$$T_{3} = \frac{\sum_{j=5}^{H} a_{j}}{\sum_{h=1}^{H} a_{h}}$$
 (upper partials) (9)

the timbre of the sound can now defined, using these relations, fixing two of these indexes; generally T_2 , T_3 are the control parameters and T_1 is calculated according the relation:

$$T_1 = 1 - T_2 - T_3$$
 (10).

From the definition of the three tristimulus index T_1 , T_2 , T_3 derives that they are all defined in the (0, 1) interval. The relative intensities of the three bands can be plotted on a two-dimensional triangular diagram, each corner of the diagram represents the total concentration of energy in a particular band, Figure 4 shows the layout of the tristimulus diagram.

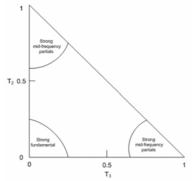


Figure 4. Tristimulus diagram.

A polyphonic synthesizer, called TIMBRALIZER-2D, has been created on a additive synthesis approach in witch the amplitude of the harmonic components are controlled according with the tristimulus model. The timbre of the generated sound can be easily controlled fixing the value of the two control parameters T_2 , T_3 , that geometrically is equivalent to fix a point inside the tristimulus triangular domain shown in Figure 4.

For our purpose the triangular domain in with is defined the tristimulus model of the timbre is rather limiting, for this reason a mapping to transform a square domain into a triangular one suitable for controlling the tristimulus parameters has been defined. Figure 5 shows a schematic representation of the mapping that has been used.

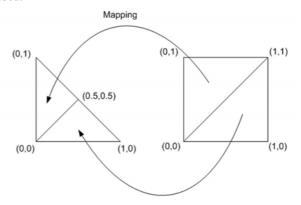


Figure 5. A schematic representation of the mapping between a square domain and a triangular domain.

By using this transformation of domain it is possible to easily control the timbre of the generated sound through any two-dimensional dynamical system. Both the Henon map, used by Witten [17], and the predatorprey model, used by Pressing [14], has been tested to control the timbre evolution. Also in this case, the synthesizer has been provided of a MIDI interface to give to the user a Note control through a external keyboard.

As in the one-dimensional case the tests we have performed show that the instrument allows to follow by an auditory point of view the evolution of the dynamical system. In this case the topologically different behaviors provided by the dynamical systems are more easy to recognize because this instrument is characterized by a wider range of possible timbres than the one-dimensional above mentioned.

This wide range of "tone-color" that can be generated by this instrument allows to obtain sounds with rich dynamical timbre with an evolution resembling the complexity of sounds generated by natural instruments. Moreover it is possible to simulate, with very interesting results, the generation of an attack transient in the sound by imposing a re-initialization of the dynamical system every time a note-on MIDI event is generated by the keyboard.

3.2.2.Morphing approach

By extending the one-dimensional application shown before it is possible to create a two-dimensional system allowing controlling the spectral content of the generated sound. We have modified the instrument above mentioned in order to add a second control parameter and then realize a two-dimensional control of the timbre of the generated sound.

In the first stage of this work we have chosen to try manipulating only the spectral energy distribution by changing the amplitude of the partials used to synthesize the sounds. Therefore we have focus our attention on the possibility to use a sound morphing between different waveform. Clearly as Wessel [16] pointed out it is possible to use many other sound characteristics, like the attack rate or the extent of synchronicity among the various components, to add control dimensions to the timbre's model adopted for the sound synthesis.

The mapping accomplished for the one-dimensional instrument has been changed in order to have two independent parameters controlling the amplitude of all the partials in the generated sound. The mapping allows to create a sound morphing between three well defined waveform: the two waveforms used in the one-dimensional case and the triangular waveform. Figure 6 shows the diagram of the mapping we have adopted.

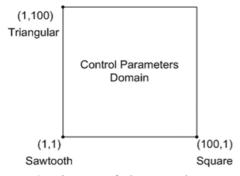


Figure 6. Diagram of the control parameters domain

The triangle waveform can be generated using a series of cosinusoidal harmonics with this amplitude distribution:

$$a_k = \begin{cases} 0 & \text{if k even} \\ \frac{1}{k^2} & \text{if k odd} \end{cases}$$
 (11)

where k = [1, ..., n] is the number of the partial; we have chosen this waveform because it is characterized by a very different distribution of the partials' amplitude compared with the other two waveform we have used. For this reason this waveform have a much mellow sound that easily allows to distinguish it from the others.

In this case the mapping equation (2) can be written as:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ y_{11} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ \cdot & \cdot \\ \cdot & \cdot \\ a_{111} & a_{112} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ \cdot \\ \cdot \\ b_{11} \end{bmatrix}$$
(12)

where $[x_1, x_2]$ is the control parameters' vector (both the control parameter are defined in the range (1, 100) and the Y vector contains the variables associated to the ten partials' amplitudes and the phase of the oscillators. Imposing the three conditions mentioned above it is possible to find the values of the elements of the vectors A and B by resolving the system of 33 equations in the 33 unknown variables. By this way it is possible to find eleven equations relating the control parameter with the synthesis parameters. The eleventh equation is used to control the phase of the oscillator that is linearly interpolated in order to have sinusoidal harmonics for the sawtooth and square waveform and cosinusoidal harmonics for the triangular waveform. In this case the synthesis parameters are represented by planes defined in the control parameters space, Figure 7 shows the plane obtained for the amplitude of the second partial.

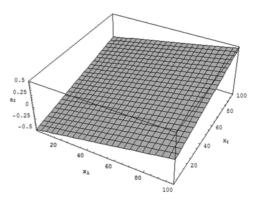


Figure 7. The plane representing the amplitude of the second partial as function of the two control parameters.

Some tests have been performed using both the twodimensional dynamical systems above mentioned to control the timbre evolution. In this case we have obtained less interesting results because by this means it is possible create a littler range of possible timbres than using the tristimulus approach. However, it is always possible recognizing in a qualitative manner the topologically different behaviors provided by the dynamical systems.

We think possible improving the performance of this system by creating a sound morphing between sounds with wide spectral differences, moreover better results can be certainly obtained by controlling in addition to the partials' amplitude also the amplitude envelope of the whole sound and the amplitude envelope of the single partials.

3.3. 3-D Application

By extending the two-dimensional application based on the morphing between waveform we have also tried to realize a three-dimensional control parameters space.

The instrument has been modified in order to add a third control parameter allowing a three-dimensional control of the timbre of the generated sound. The mapping has been changed in order to have three independent parameters controlling the amplitude of all the partials in the generated sound. The mapping allows to create a sound morphing between four well defined waveform: the waveforms used in the two-dimensional case and a pulse train waveform.

The pulse train waveform can be approximate using a series of sinusoidal harmonics with a constant amplitude distribution:

$$a_k = 1 \qquad \forall k \tag{13}$$

where k = [1, ..., n] is the number of the partial.

In this case the mapping equation (2) can be written as:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{11} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \vdots & \vdots & \vdots \\ a_{111} & a_{112} & a_{113} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_{11} \end{bmatrix}$$
(14)

where $[x_1, x_2, x_3]$ is the control parameters' vector (both the control parameter are defined in the range (1, 100)) and the Y vector contains the variables associated to the ten partials' amplitudes and the phase of the oscillators. Imposing the four conditions mentioned above it is possible to find the values of the elements of the vectors A and B by resolving the system of equations. By this way it is possible to find the equations relating the control parameter with the synthesis parameters. In this case the synthesis parameters are represented by hyperplanes defined in the control parameters space.

We have performed some tests using threedimensional systems like the Lorenz's system [2] and the Chua Oscillator [3]. In this case, the obtained results are less interesting than in the previous cases because it is more difficult recognizing the topologically different behaviors provided by the dynamical systems.

For this kind of application we need some more work to do in order to define a mapping strategy to control independently many sound characteristics, like the amplitude envelope or the extent of synchronicity among the various components, avoiding the limiting assumption of a timbre control based only on the modification of the spectral energy distribution.

4. CONCLUSION

A new approach to use chaotic dynamical systems for controlling the parameters of a sound synthesis process has been presented. In literature, all previous approaches can be divided in two categories: high level, finalized to melodic pattern generation, low level, in which the dynamical systems are used to generate sound samples. We have chosen to work at an intermediate level using chaotic systems to control the parameters of a sound synthesis process.

The dynamical systems have been used to control the parameters of a sound synthesis process mapping the evolution of the system into timbre modification of the generated sound. This intermediate level mapping allows the user, by using a MIDI keyboard, to play sounds evolving according to the chaotic dynamical systems controlling only the high level musical parameters, like pitch and amplitude.

The tests we have performed show that by using the instruments presented here it is possible recognizing in a qualitative manner the topologically different behaviors provided by the dynamical systems.

Some more work is needed to improve the instruments, particularly for the two and three dimensional applications based on the morphing approach.

We think possible improving the performance of these instruments by creating a sound morphing between sounds with wide timbric differences. In future work, in fact, we will try to use sounds simulating those of natural instruments that can create by an analysis resynthesis process. For this reason we will create more complex mapping strategies for controlling in addition to the partials' amplitude also the amplitude envelope of the whole sound and the amplitude envelope of the single partials.

5. ACKNOWLEDGMENT

Thanks to Agostino Di Scipio for the kind support given in the bibliographic references research and to Enrico Cupellini for the daily discussions and suggestions.

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